

# **BRANL GLM Manual**

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version 1.9E  
BRSystems,Inc.  
July 14,2023



## update history

date	version	content
2021/11/16	1.0	First edition
2021/12/7	1.1	GLM modify sample fig.
2021/12/29	1.2	GLM add "3.2 configuration setting"
2022/1/31	1.3	GLM modify the table of $\beta$ -value
2022/2/2	1.4	GLM add the procedure of consecutive calculation
2022/2/12	1.5	Add the explanation of input data
2022/3/9	1.6	Update GMF function, how to see the $\beta$ -value in the case of FIR
2022/3/31	1.7	Add the average value of $\beta$ -value in FIR_I/II
2022/11/7	1.8	Clarify the prerequisite of GLM and the "stats"
2023/7/14	1.9E	English Version

# 1. Start up

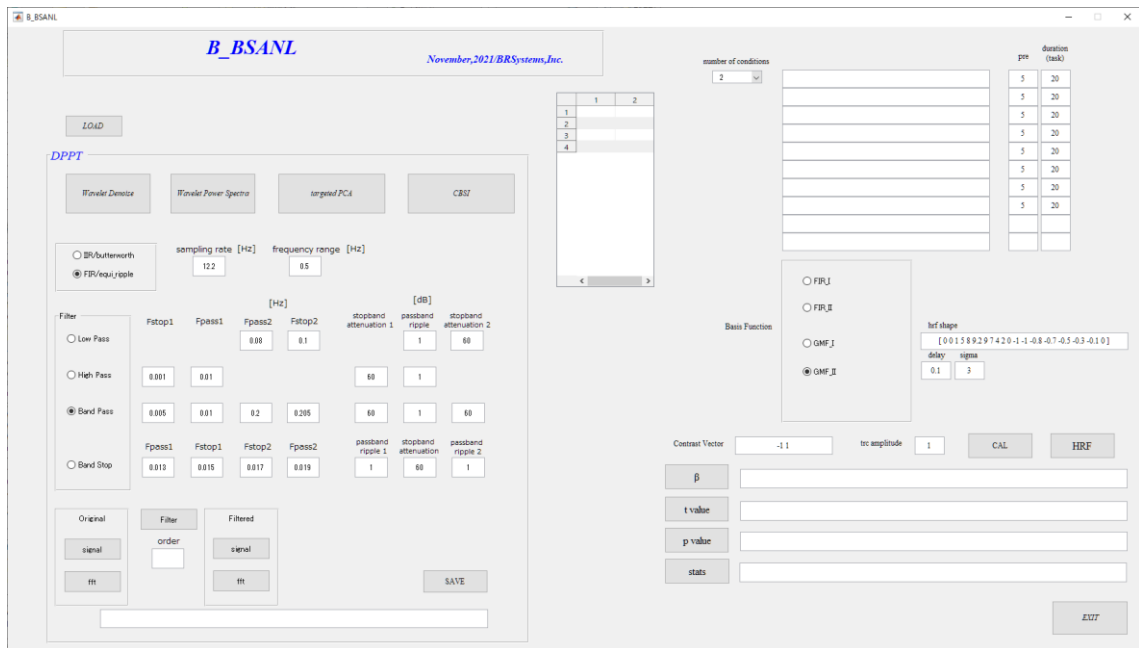
double click BRANL.exe

Next screen launches.

Three methods, WaveletDenoising, tPCA and pre-whitening are implemented for data pre-processing to reduce noise, MA.

Pre-whitening is calculated in the other tool.

## Top View



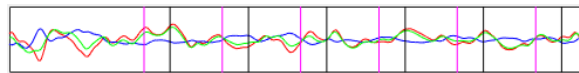
## 2. GLM

The prerequisite of GLM are,

- (1) signal is normal distribution.
- (2) the effect of outliers on signal is negligible.
- (3) the effect of serial correlation is negligible.

### 2.1 GLM Algorithm

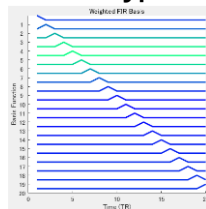
measured concentration data  $Y$  (HbO,HbR)



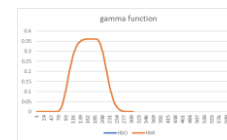
$$Y = X\beta + \varepsilon$$

basis function

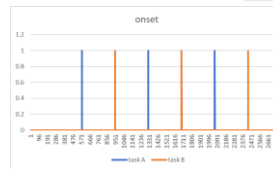
fir type



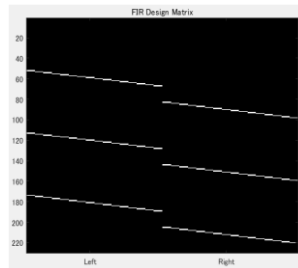
gamma function type



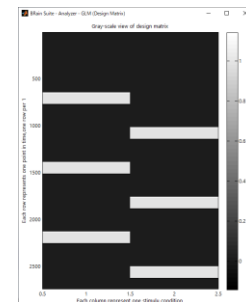
onset/duration



design matrix (convolution)



$X$

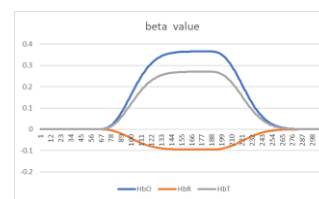
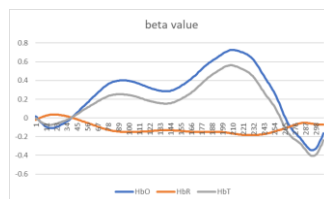


$$X^* = (X^T X)^{-1} X^T$$

$\beta$  (based on OLS)

$$\beta = X^* Y \quad (\text{MATLAB/regress})$$

HRF



t value

$$t = \frac{c^T \beta}{\sqrt{\varepsilon^2 c^2 (X^T X)^{-1} c}}$$

- \* "regress" or "robustfit" is used in GLM calculation.
- \* stats information of regress function (Matlab) is available, so you can verify the R2 score.
- \* Y:data X:regressor  $\beta$ :coefficient(weight)  $\varepsilon$ :error c:contrast vector

## 2.2 HRF Basis Function

HRF basis function used in fNIRS software is listed the below table.

HRF Basis Function				
	GAMMA		FIR	
"Functional Magnetic Response Imaging" second edition SA Huettel, AW Song, G McCarthy	Double gamma function, Gamma function, Gaussian function		Finite Impulse Response (FIR)	
SPM (NIRS_SPM)	Double gamma function spm_hrf.m, spm_Gpdf.m			
Homer3	Modified Gamma function convolved with square wave of duration T	Modified Gamma function and its derivative convolved with square wave of duration T	Consecutive sequence of gaussian function	
AnalyzIR(MDPI)	Canonical HRF Double gamma function	Gamma Function	FIR-Deconvolution	FIR-Impulse Response Deconvolution
BSANL	GMF_I: Double gamma function $gampdf(y, s1, r1)$ $-k \times gampdf(y, s2, r2)$ $k: coefficient(ex. 0.5)$	GMF_II: Gamma Function (1) $gampdf(y, s, r) \dots$ $p(y s, r) = \frac{r^s}{\Gamma(s)} y^{s-1} exp(-ry)$ $y: time, s: shape(peak time),$ $r^{-1}: scale(dispersion time)$	FIR_I Consecutive 1[sec] BoxCar	FIR_II Consecutive gaussian function

(1) page138, 「データ解析のための統計モデリング入門」、久保拓弥

\* Second term of Gamma function means the undershoot.

### •Four Methods are provided as basis function.

(1) FIR\_I

Flexible model

Multiple boxcar function duration 1[sec] are used as regressor.

(2) FIR\_II

Flexible model

Consecutive Gaussian function is used as regressor.

(3) GLM\_I

Fixed canonical shape

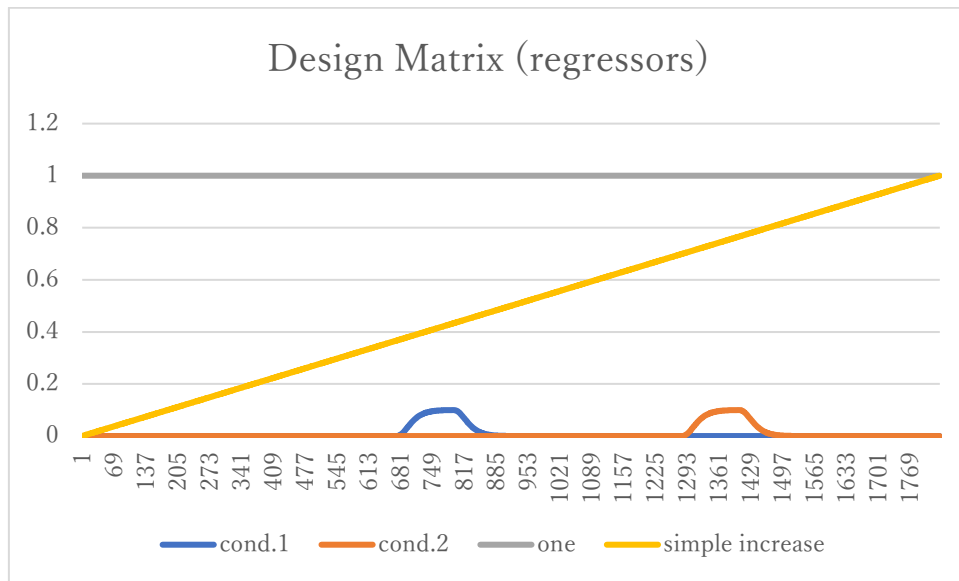
Double gampdf function (Matlab)

(4) GLM\_II

Fixed canonical shape

Single gampdf function (Matlab)

•To compute coefficient estimates for a model with terms, include a column of ones and a column of simple increase drift [0,1] as regressors.



### 3. Configuration

Input data is limited to Concentration data (HbO,HbR).

Data conversion. raw(electric signal) to OD, OD to Concentration is done by BBrain Analyzer.

File format of input data is csv.

#### 3.1 Configuration setting

\*Load data(csv file)

## [BRANL Configuration]

The screenshot shows the BRANL Configuration interface with the following elements and callouts:

- 1**: A table for task timing.
 

	1	2
1		
2		
3		
4		
- 2**: A dropdown menu for "number of conditions" set to 1.
- 3**: A list of 8 conditions, each with a corresponding empty input field for onset time.
- 4**: Input fields for "pre" (5) and "task" (20) durations.
- 5**: Radio buttons for "Basis Function" selection: FIR\_I, FIR\_II (selected), GMF\_I (Double Gamma), and GMF\_II (Single Gamma).
- 6**: Parameters for FIR\_I: average (1), variance (1).
- 7**: Parameters for FIR\_I: s1 (6), r1 (1), s2 (10), r2 (1).
- 8**: Parameters for FIR\_I: k (0.5), stimulus duration (10).
- 9**: Contrast Vector input field containing "-1 1".
- 10**: trc amplitude input field containing "1".
- 11**: Buttons for "CAL" and "HRF".
- 12**: Buttons for "CAL" and "HRF".
- 13**: Input field for  $\beta$ .
- 14**: Input field for t value.
- 15**: Input field for p value.
- 16**: Input field for stats.
- 17**: An "EXIT" button.

- ① start and end time of each task [sec]  
this time is displayed for reference only.
- ② number of conditions
- ③ onset time of task  
onset time is configured per each condition
- ④ pre-task range [sec], task duration range [sec]
- ⑤ select GLM method
  - Ⓒ FIR\_I

◎FIR\_II

◎GMF\_I

canonical hrf, double gampdf function (MATLAB)

$$hrf = gampdf(y, s1, r1) - k \times gampdf(y, s2, r2)$$

*k: coefficient(ex. 0.5)*

*y: time, s: shape(peak time),  $r^{-1}$ : scale(dispersion time)*

Default value of s1,r1,s2,r2,k are set by reference to "fMRI in Neuroscience: Modeling the HRF with FIR Basis Functions, The OG Clever Machine"

◎GMF\_II

canonical hrf, single gampdf function (MATLAB)

*gampdf(y, s, r) ...*

$$p(y|s, r) = \frac{r^s}{\Gamma(s)} y^{s-1} \exp(-ry) \quad (*)$$

*y: time, s: shape(peak time),  $r^{-1}$ : scale(dispersion time)*

(\*) page138, "Introduction to statistical modeling for data analysis", T.Kubo, Iwanami Shoten, Publishers

Parameters are set depending on the measured data.

- ⑥ mean and variance of Gamma function, default is 1,1.ガンマ関数の係数
- ⑦ *s: shape(peak time),  $r^{-1}$ : scale(dispersion time) k: coefficient(ex. 0.5)*
- ⑧ stimulus duration time [sec]
- ⑨ contrast vector
- ⑩ amplitude factor of task related component
- ⑪ run GLM calculation
- ⑫ display HRF figure
- ⑬ save the result of  $\beta$ -value
- ⑭ save the result of t-value
- ⑮ save the result of p-value
- ⑯ save the result of stats-value



stats includes  $R^2$  statistic, F-statistic, p-value, an estimate of the error variance

⑰ Exit BRANL

[onset configuration]

- ① these numerics for reference only
- ② number of load condition
- ③ configure onset time by task condition

The screenshot shows the BRANL onset configuration interface. It includes a table for reference numerics, a dropdown for the number of conditions, a table for configuring onset times by task condition, a Basis Function selection menu, and various statistical output fields.

	start	end
1	52	62
2	83	93
3	113	123
4	144	154
5	174	184
6	204	215

number of conditions: 3

	pre	task
1	30	70 170
2	80	110 200
3	150	
4		
5		
6		
7		
8		

Basis Function:  FIR\_II

sigma: 3      boncar stimulus duration: 10

Contrast Vector: -1 1      trc amplitude: 1      CAL      HRF

$\beta$       \_\_\_\_\_

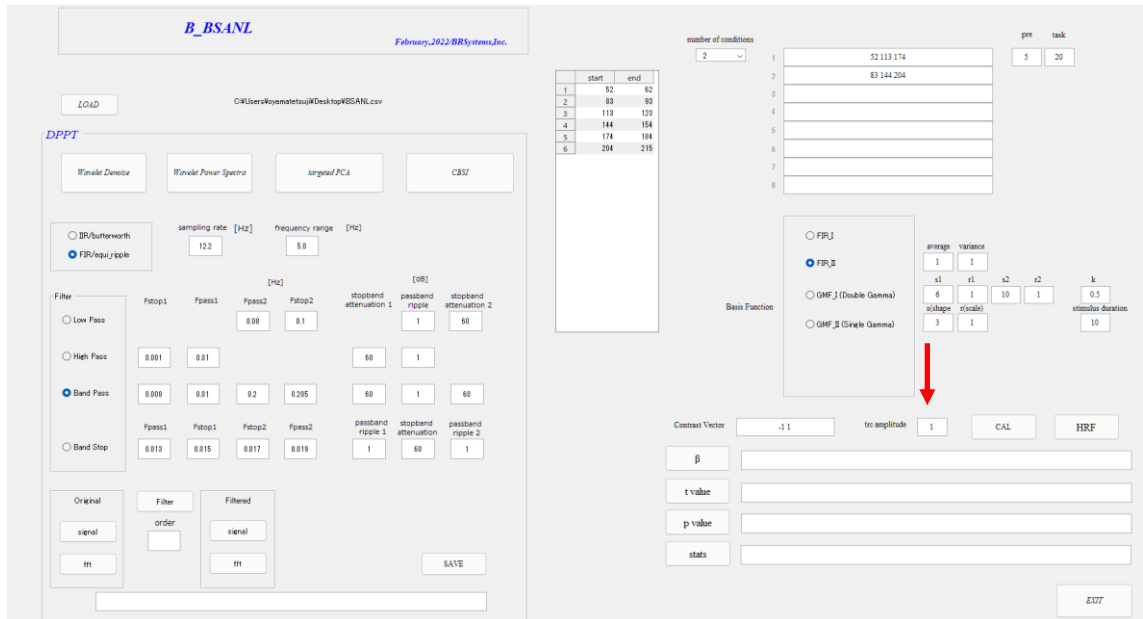
t value      \_\_\_\_\_

p value      \_\_\_\_\_

stats      \_\_\_\_\_

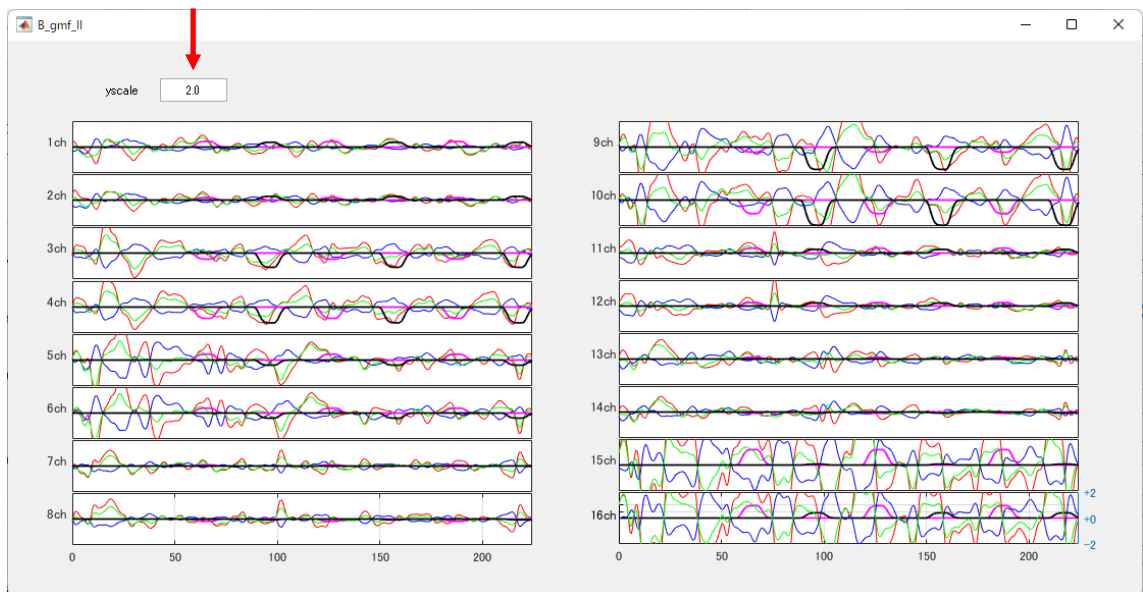
EXIT

[amplitude factor of task related component]



trc amplitude set 1, and click "CAL", then figure below displayed.

data scale will be changed, according to the value "yscale"

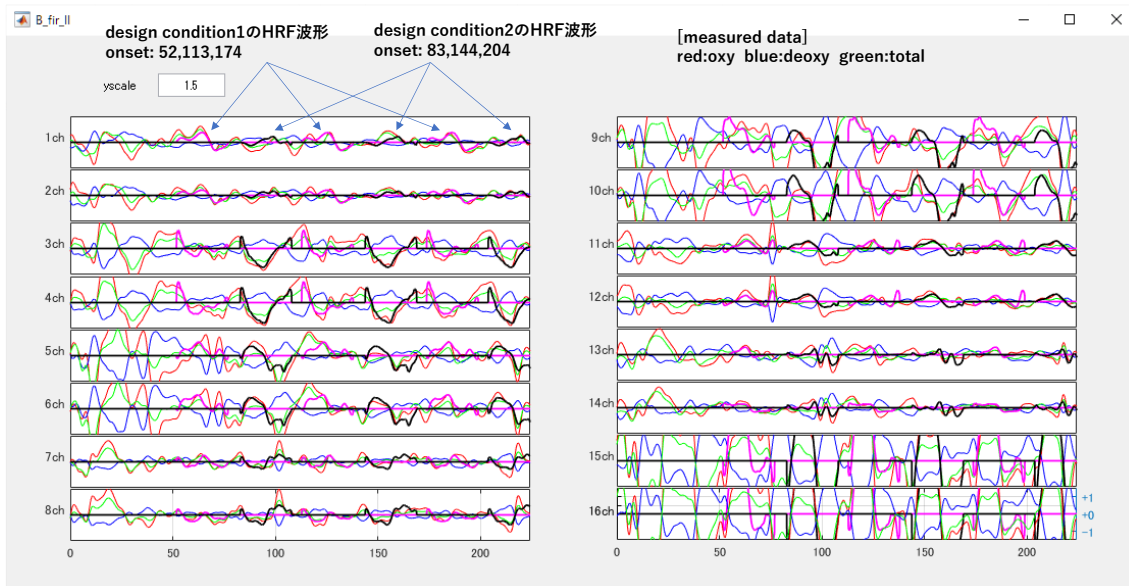


The color of task related component  
 condition 1: magenta, condition 2: black, condition 3: cyan,  
 condition 4: magenta, condition 5: black, condition 6: cyan,  
 - - -

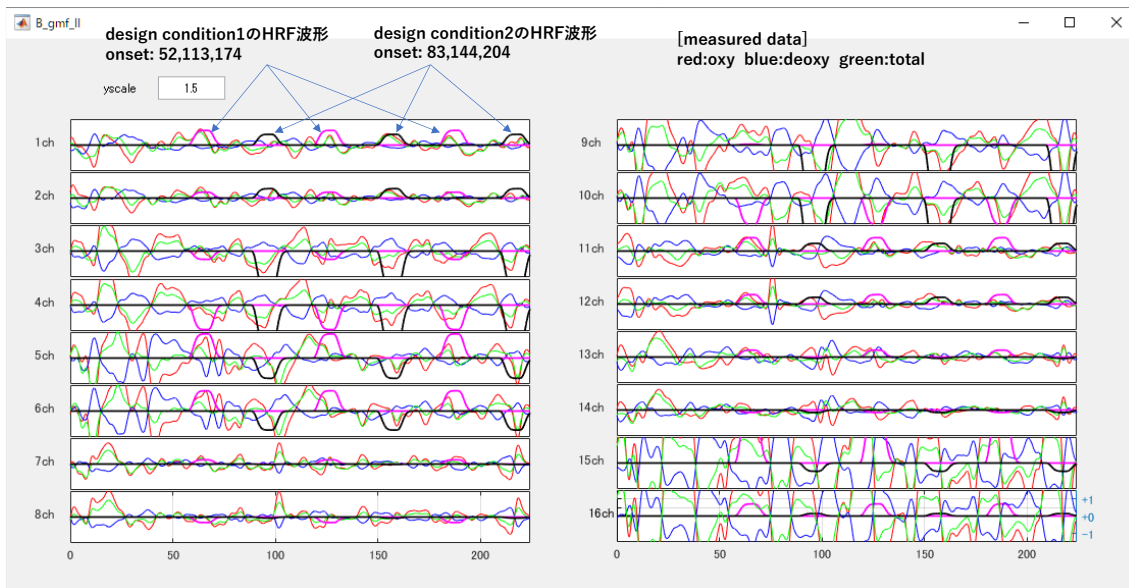
### 3.2 explanation of figure

#### concentration signal and task related component

#### FIR\_II example



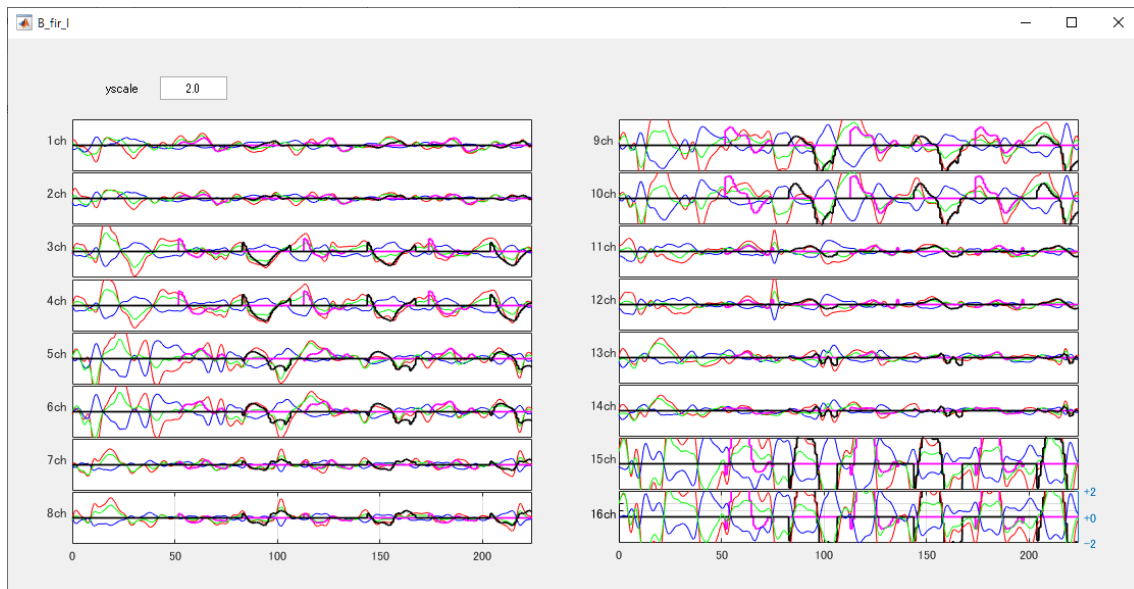
#### GMF\_II example



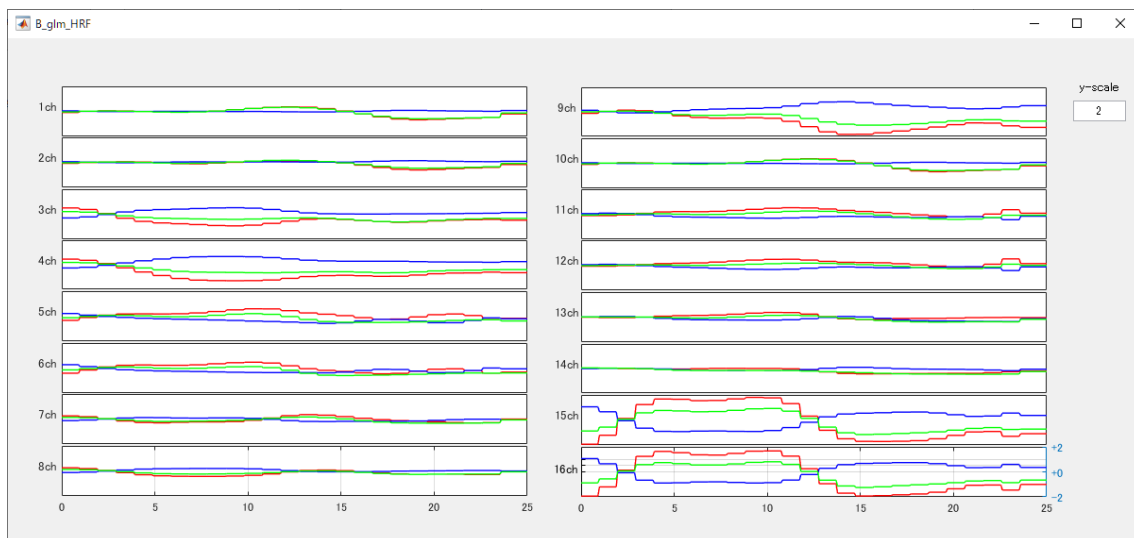
#### 4 Figure for each GLM method

##### (1) FIR-I

HbO/R/T and task related component

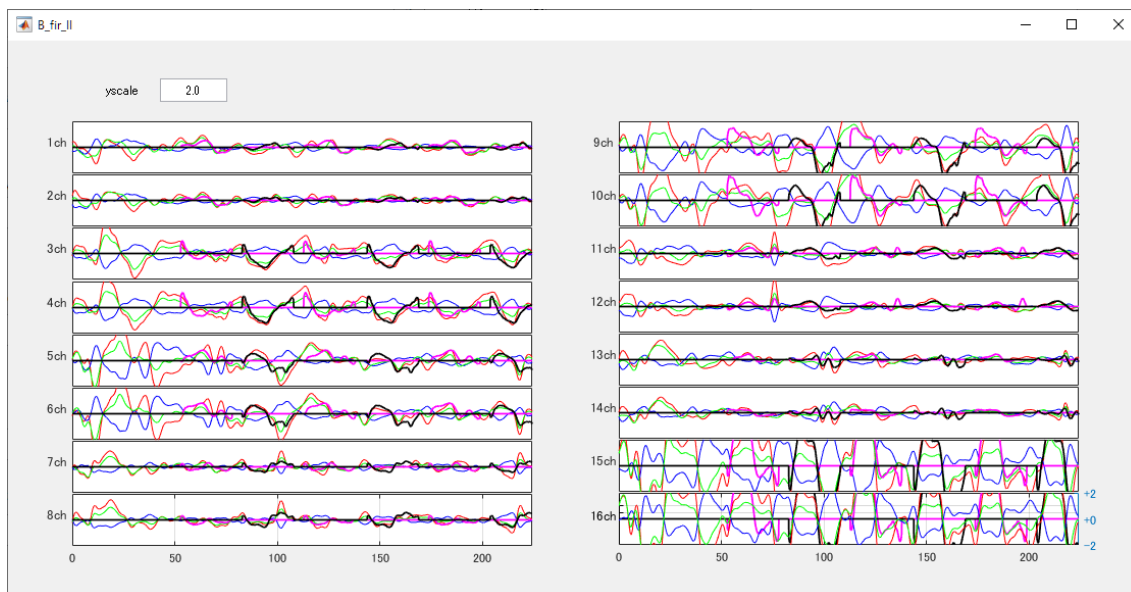


HRF figure

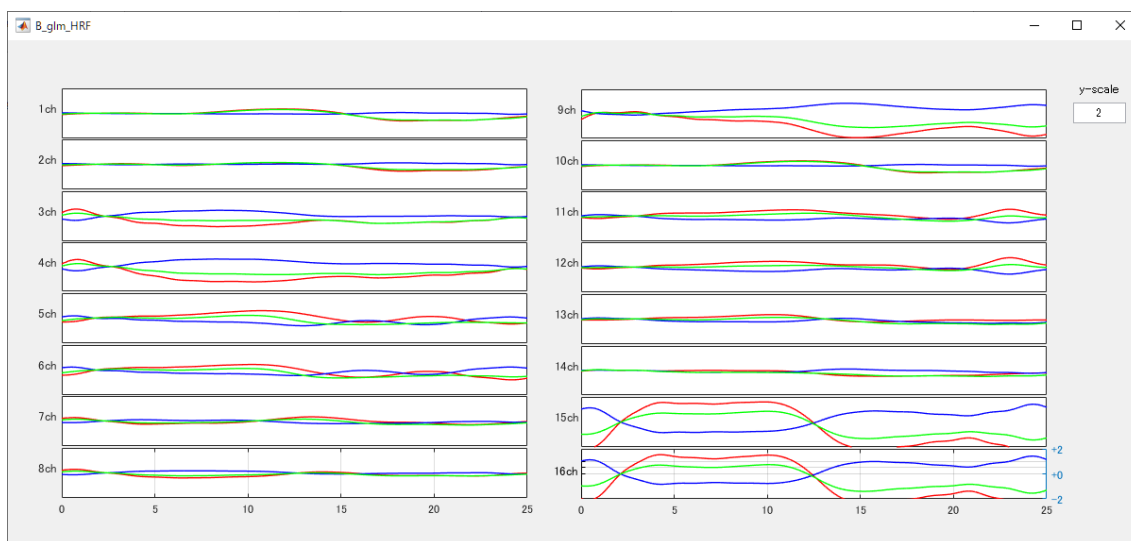


## (2) FIR-II

HbO/R/T and task related component

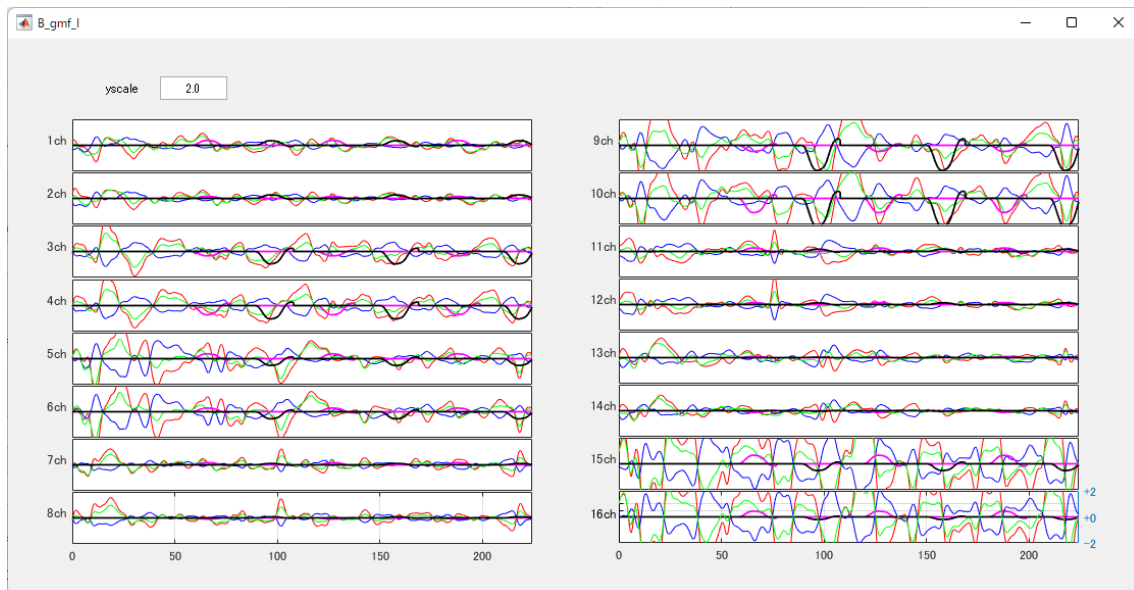


HRF figure

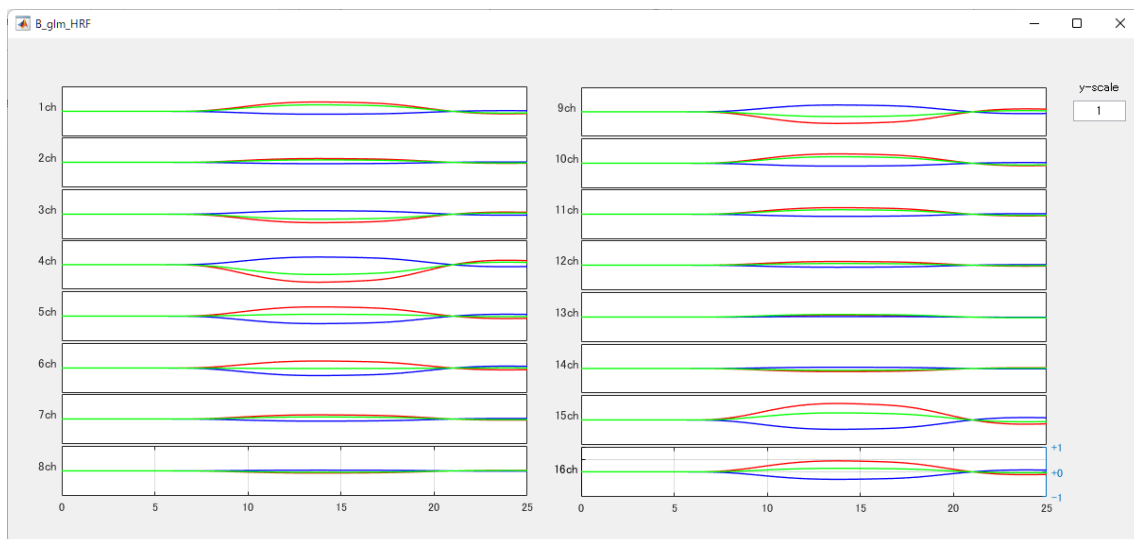


### (3) GMF-I

HbO/R/T and task related component

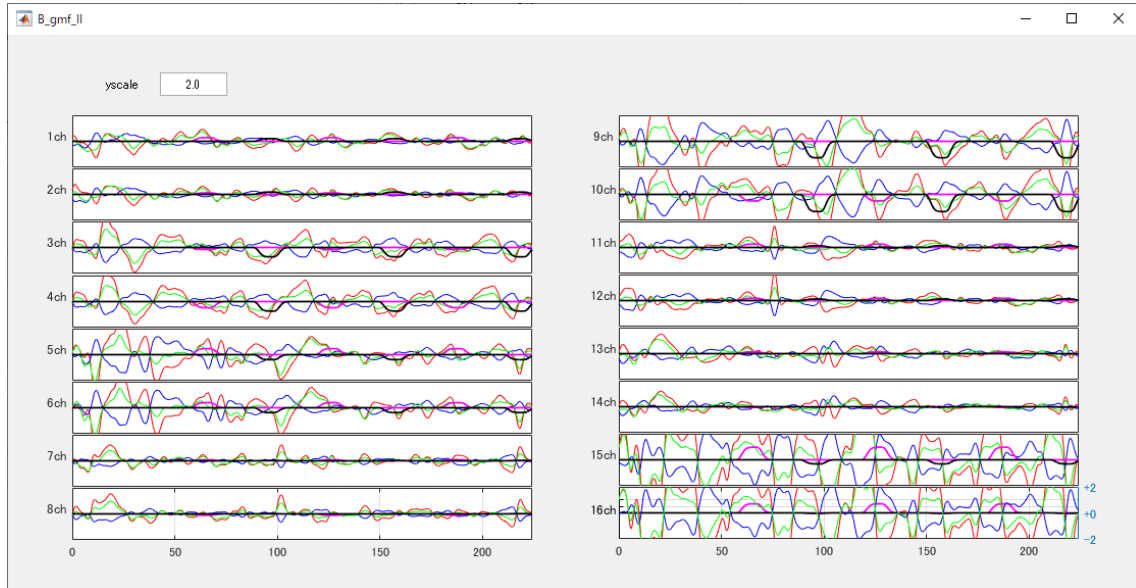


HRF figure

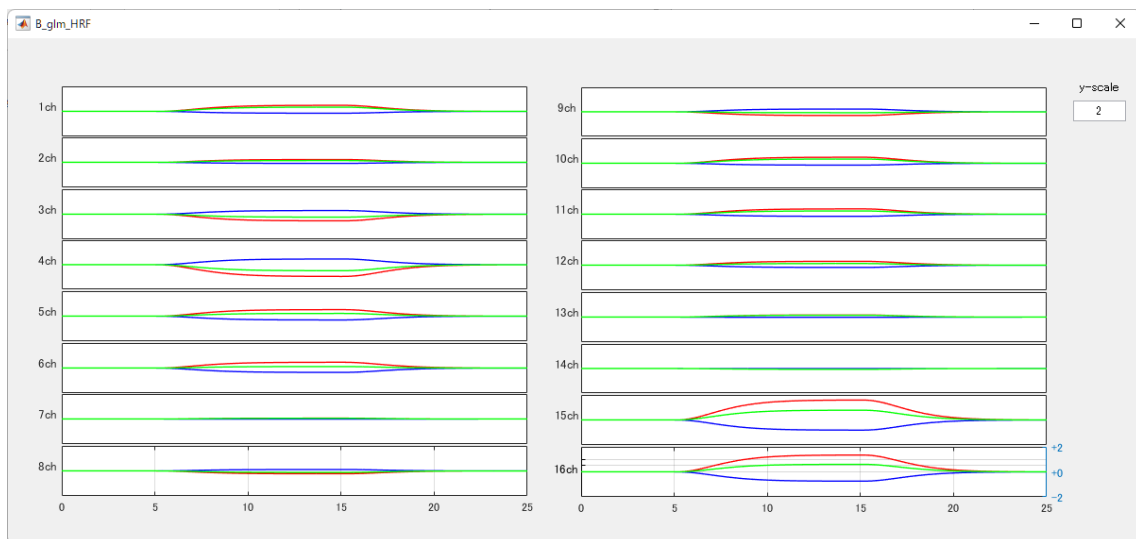


#### (4) GMF-II

HbO/R/T and task related component



HRF figure



## 5 Result (sample)

### (1) $\beta$ -value (GMF)

例 : HbO/HbR  $\beta$ -value of load condition 1 and load condition 2

beta-value	ch1	ch2	ch3	ch4	ch5	ch6	ch7	ch8	ch9	ch10	ch11	ch12	ch13	ch14	ch15	ch16
1_HbO	0.813	0.339	-0.458	-1.326	1.312	1.114	0.219	-0.305	0.096	-1.354	0.73	0.517	0.387	-0.106	2.849	2.1
1_HbR	-0.247	-0.137	0.237	0.606	-0.692	-0.783	-0.096	0.12	0.169	0.743	-0.272	-0.312	-0.035	0.03	-1.262	-1.047
2_HbO	0.571	0.483	-2.037	-2.082	-1.016	-0.979	0.036	0.001	-3.435	-3.58	0.4	0.355	-0.028	-0.087	-0.638	0.131
2_HbR	-0.168	-0.136	1.121	1.094	0.053	0.262	0.03	0.039	1.309	1.511	-0.167	-0.137	-0.108	-0.158	-0.647	-0.759

### $\beta$ -value(FIR)

[sec]	HbO																HbR		
	Cnd	ch1	ch2	ch3	ch4	ch5	ch6	ch7	ch8	ch9	ch10	ch11	ch12	ch13	ch14	ch15	ch16	ch1	ch2
condition one	1	0.25595	0.12409	0.63993	0.69447	-0.27415	-0.09553	0.2507	0.16634	1.1441	1.7563	0.044129	-0.04143	0.033447	0.21729	-0.17607	-0.23086	-0.0574	-0.0327
	2	0.052785	0.034659	-0.27287	-0.31243	0.44022	0.33892	-0.12538	-0.10055	-0.48197	-0.86224	-0.05849	-0.01954	-0.08542	-0.15483	0.11633	0.18082	-0.04096	-0.02373
	3	0.29775	0.17029	0.35694	0.49968	-0.11258	0.15365	0.12278	0.050535	1.4558	2.0505	0.077554	-0.01289	0.023451	0.22276	1.4447	1.3863	-0.07113	-0.05884
	4	0.052831	0.088236	-0.59828	-0.57579	0.46242	0.43203	-0.27368	-0.23479	-0.3461	-0.77207	0.077664	0.077711	-0.02568	-0.14139	1.3186	1.3839	-0.04056	-0.05862
	5	0.22061	0.12718	0.017751	0.10584	-0.02872	0.2047	-0.06134	-0.15287	0.97864	1.2506	0.20414	0.072109	0.076948	0.1463	1.7158	1.6378	-0.05711	-0.05055
	6	0.043863	0.02649	-0.55024	-0.68568	0.34535	0.34569	-0.25653	-0.28733	-0.31441	-0.68828	0.11057	0.080053	0.037349	-0.10096	1.2159	1.1678	-0.03963	-0.03139
	7	0.15784	0.046651	-0.12389	-0.25759	0.05154	0.22087	-0.01596	-0.16904	0.65423	0.7437	0.13097	0.065344	0.060129	0.13394	1.4907	1.3723	-0.04144	-0.01443
	8	0.095342	0.02125	-0.51142	-0.68805	0.3336	0.35136	-0.18524	-0.29097	-0.17712	-0.51652	0.15092	0.1543	0.054441	-0.01096	1.2842	1.1948	-0.05158	-0.02233
	9	0.20116	0.10549	-0.28665	-0.34782	0.20658	0.32688	-0.07353	-0.1925	0.50915	0.47035	0.18911	0.14825	0.11867	0.079814	1.5928	1.4899	-0.04658	-0.03182
	10	0.20805	0.095439	-0.51124	-0.63098	0.41116	0.4135	-0.17438	-0.24732	0.001515	-0.29095	0.25097	0.22204	0.14625	-0.00518	1.4594	1.4117	-0.06439	-0.03431
	11	0.27073	0.14081	-0.26436	-0.40367	0.3418	0.40574	0.002369	-0.1733	0.44736	0.35077	0.23368	0.17978	0.17676	0.085084	1.6645	1.6303	-0.02631	-0.0094
	12	0.29729	0.15135	-0.36561	-0.58341	0.40097	0.37188	0.000736	-0.16801	-0.12244	-0.3966	0.33378	0.27589	0.16292	0.005294	1.2097	1.2057	-0.07208	-0.03921
	13	0.31703	0.15425	-0.1169	-0.29531	0.22013	0.20789	0.18824	0.008731	-0.02951	-0.19128	0.21148	0.094023	0.069428	-0.00781	0.71236	0.68979	-0.04103	-0.007
	14	0.28263	0.13726	-0.18546	-0.41309	0.14907	0.055826	0.10545	-0.03968	-0.57259	-0.84163	0.25621	0.17857	-0.04361	-0.15318	-0.09262	-0.18942	-0.10316	-0.06068
	15	0.20027	0.11056	-0.10246	-0.25444	0.23174	0.20392	0.10677	0.020199	-0.12714	-0.35944	0.064199	-0.00488	-0.11291	-0.13251	0.31208	0.20699	-0.02094	-0.01137
	16	0.14048	0.054491	-0.28922	-0.399	0.061164	-0.01151	-0.03049	-0.11778	-0.57412	-0.87507	0.25799	0.24571	-0.09302	-0.16538	-0.18277	-0.28273	-0.10853	-0.07424
	17	-0.02763	-0.0477	-0.14841	-0.2777	0.077765	0.1104	0.022347	0.02147	0.069453	-0.11687	-0.04366	-0.0707	-0.03248	-0.08939	0.47194	0.41655	0.037941	0.040013
	18	-0.0458	-0.11117	-0.40534	-0.50773	-0.12441	-0.16047	-0.15185	-0.25021	-0.55025	-0.71537	0.2406	0.25837	-0.01908	-0.17274	-0.33359	-0.3829	-0.11077	-0.05268
	19	-0.21654	-0.18752	-0.19331	-0.23331	0.14926	0.22304	-0.01065	-0.06189	0.33277	0.22502	-0.23279	-0.28321	-0.05219	-0.07886	0.67178	0.62896	0.11671	0.1083
	20	-0.04527	-0.13856	-0.38574	-0.44956	0.029786	-0.0407	-0.20828	-0.31435	-0.61078	-0.80263	0.32171	0.33929	-0.04252	-0.17273	-0.43395	-0.51914	-0.16455	-0.08847
	21	-0.16419	-0.12771	-0.04194	-0.05718	0.41624	0.49028	0.006673	0.047896	0.73919	0.52421	-0.46577	-0.53759	-0.00862	-0.05484	1.1951	1.1727	0.1356	0.11226
	22	0.035934	-0.0632	-0.39399	-0.47603	0.013488	-0.10904	-0.31632	-0.38505	-0.66521	-0.80909	0.4407	0.45606	-0.00042	-0.20874	-0.34447	-0.41271	-0.23909	-0.15621
	23	-0.17985	-0.09388	-0.00306	-0.06393	0.27071	0.35159	-0.20009	0.037012	0.84367	0.75052	-0.39048	-0.46954	-0.01338	-0.00667	-0.3973	1.3614	0.17843	0.12714
	24	0.063193	-0.04924	-0.39321	-0.42004	-0.16846	-0.28229	-0.26654	-0.37046	-0.84681	-0.84217	0.92756	1.0722	-0.00526	-0.0985	-0.65175	-0.79211	-0.22884	-0.14264
condition two	1	0.30052	0.17544	0.34157	0.34719	-0.20273	-0.33209	0.35462	0.26702	-0.23195	0.10834	0.29355	0.20923	0.11778	0.071027	-1.783	-1.6316	0.005479	0.012497
	2	-0.19578	-0.10462	-0.44206	-0.30287	0.4614	0.60886	-0.29001	-0.27013	0.48972	0.38248	-0.08086	-0.02273	-0.12235	-0.08811	1.6883	1.6792	0.056144	0.031691
	3	0.27312	0.16141	0.026182	-0.02048	0.0049	-0.05478	0.22912	0.075073	-0.09722	0.34096	0.40105	0.27181	0.087767	-0.00465	-0.57554	-0.3137	0.013076	0.017247
	4	-0.22486	-0.14847	-0.70638	-0.68043	0.47182	0.65067	-0.36929	-0.44394	0.4467	0.39904	0.009218	0.001312	-0.14123	-0.15822	2.5264	2.6029	0.072414	0.059029
	5	0.14778	0.048825	-0.26384	-0.37107	0.086042	0.064357	0.02394	-0.11338	-0.0825	0.15755	0.36642	0.23017	0.040732	-0.06331	0.47864	0.6774	0.028631	0.046505
extra	0	-0.12381	0.019769	0.22341	0.26892	0.005066	-0.08776	0.05525	0.24084	0.054239	-0.03377	-0.0898	-0.03618	0.10629	0.077613	-0.45243	-0.49918	0.040767	-0.01419
	0	-0.08707	-0.15179	0.53703	0.62591	-0.22244	-0.1515	0.036554	-0.06093	0.32254	0.60729	-0.23953	-0.23787	-0.11774	0.050136	-0.76457	-0.80537	0.013415	0.052499
mean SD	t1_mean	0.086028	0.011845	-0.2775	-0.39182	0.17987	0.18289	-0.06754	-0.15661	-0.05227	-0.2306	0.15725	0.12653	0.021181	-0.06075	0.66519	0.59841	-0.04686	-0.02045
	t1_sd	0.16623	0.11106	0.16499	0.18244	0.17572	0.21984	0.13884	0.14085	0.51985	0.57405	0.30456	0.34523	0.08402	0.097306	0.79541	0.81107	0.10933	0.076152
	t2_mean	0.099904	0.06015	-0.39518	-0.43127	-0.06913	-0.04093	-0.01741	-0.05656	-0.43552	-0.45717	0.09201	0.083586	-0.07094	-0.09336	0.19624	0.3212	-0.01397	-0.00194
	t2_sd	0.10306	0.11356	0.23624	0.29745	0.4699	0.42767	0.22816	0.27655	0.73698	0.60375	0.30039	0.21424	0.26111	0.17252	1.2666	1.1633	0.087516	0.10436

- each row corresponds to 1[sec].
- In this case, as Pre=5[sec]、Task=20[sec]、first 5 rows is Pre parts, next 19 rows are Task parts.



## (2) t-value

example of contrast vector [-1 1]

t test

t-value	ch1	ch2	ch3	ch4	ch5	ch6	ch7	ch8	ch9	ch10	ch11	ch12	ch13	ch14	ch15	ch16
HbO	-2.939	2.17	-10.023	-4.496	-12.29	-10.796	-2.097	3.027	-12.744	-7.854	-3.602	-1.846	-4.58	0.252	-7.101	-4.032
HbR	1.781	0.03	11.877	6.145	6.591	9.742	3.181	-1.675	7.604	5.234	1.6	2.845	-1.12	-3.417	2.526	1.196

## (3) p-value

example of contrast vector [-1 1]

p-value

p-value	ch1	ch2	ch3	ch4	ch5	ch6	ch7	ch8	ch9	ch10	ch11	ch12	ch13	ch14	ch15	ch16
HbO	0.001661	0.015032	0	3.60E-06	0	0	0.018039	0.001246	0	2.89E-15	0.000161	0.032485	2.43E-06	0.40057	7.85E-13	2.84E-05
HbR	0.03751	0.48801	0	4.58E-10	2.61E-11	0	0.000742	0.047022	1.95E-14	8.91E-08	0.054858	0.002239	0.1314	0.000321	0.005804	0.11583

## (4) stats-value (ref.8)

stats	ch1	ch2	ch3	ch4	ch5	ch6	ch7	ch8	ch9	ch10	ch11	ch12	ch13	ch14	ch15	ch16
1_R2	0.28594	0.24456	0.27783	0.33631	0.16254	0.19182	0.25582	0.27869	0.30366	0.34864	0.23257	0.19938	0.16804	0.22046	0.43985	0.4383
1_F	23.9348	18.3327	23.5886	30.8727	10.9906	13.5215	19.8776	24.9492	25.1723	30.6976	17.7358	14.2571	12.0596	16.8537	45.1314	45.0887
1_p(F)	5.39E-168	4.10E-128	1.34E-165	3.11E-214	2.19E-72	5.26E-92	2.40E-139	5.73E-175	1.72E-176	4.17E-213	1.00E-123	1.24E-97	1.02E-80	3.41E-117	1.23E-299	2.13E-299
1_ervar	0.10957	0.071773	0.41188	0.44255	0.64665	0.64602	0.11584	0.14927	1.2114	1.2087	0.13928	0.13062	0.14036	0.091862	2.8269	2.7689
2_R2	0.10556	0.078911	0.31822	0.35738	0.14523	0.19551	0.27626	0.27522	0.22685	0.30456	0.24276	0.22639	0.20272	0.21477	0.39029	0.42359
2_F	7.0252	4.8929	29.0418	34.287	9.4675	13.6547	23.1084	25.6012	17.1845	25.368	18.5259	16.7273	15.0333	15.9504	36.6452	42.5171
2_p(F)	3.60E-41	1.24E-24	2.35E-202	8.54E-236	1.99E-60	5.02E-93	2.93E-162	2.07E-179	1.19E-119	7.97E-178	1.58E-129	2.98E-116	1.57E-103	1.92E-110	2.94E-250	7.53E-285
2_ervar	0.037015	0.032997	0.090099	0.097522	0.22831	0.19717	0.023392	0.034632	0.3733	0.33431	0.067316	0.060732	0.070003	0.048399	0.7554	0.69201

- 1\_xx:HbO, 2\_xx:HbR
- $R^2$ -statistics : coefficient of determination, The proportionate amount of variation in the response variable y explained by the independent variables X.  $R^2 = 1 - \frac{SSE}{TSS}$  SSE: error sum of squares  
TSS: total sum of squares
- F-statistics :  
$$F = \frac{RSS/(p - 1)}{s^2}$$
  
RSS: regression sum of squares  
p: number of variables, s<sup>2</sup>: estimator of error variance
- p-value : the probability of observing  $R^2$  value or larger
- ervar : the estimator of error variance as the square of the RMSE value

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